

Color Magnetic Corrections to Quark Model Valence Distributions

C.J. Benesh, T. Goldman,
Theoretical Division, MSB283,
Los Alamos National Laboratory,
Los Alamos, NM 87545,
and,
G.J. Stephenson Jr.,
Physics Division, MSD434,
Los Alamos National Laboratory,
Los Alamos, NM 87545

June 5, 2005

Abstract

We calculate order α_s color magnetic corrections to the valence quark distributions of the proton using the Los Alamos Model Potential wavefunctions. The spin-spin interaction breaks the model SU(4) symmetry, providing a natural mechanism for the difference between the up and down distributions. For a value of α_s sufficient to produce the $N - \Delta$ mass splitting, we find up and down quark distributions in reasonable agreement with experiment.

1 Introduction

Historically, quark models have provided us with a convenient, if simplistic, method for making quantitative calculations of low energy hadronic observ-

ables. Unfortunately, much of the available data on hadronic structure is taken in an energy regime well beyond that at which simple quark models are expected to be valid. In particular, the distribution of quarks in the nucleon as a function of their light cone momentum fraction has been measured in numerous experiments over a wide energy range in the scaling regime. In order to take advantage of this data, it was necessary to invent an argument allowing an extrapolation from the low energy quark model regime to the high energies where direct measurements can be performed.

Such an argument was put forward by Jaffe and Ross[1] in 1980. Quark models, they argued, could only be a representation of QCD at a relatively low renormalization scale, $\mu^2 \approx 1/R_{proton}^2$, as a result of their relative simplicity. More explicitly, they argued that, at large μ^2 , experiments are able to resolve structures of order $1/\mu$, and consequently one sees a hadron as a very complicated object, composed of valence quarks, lots of glue, and a sizeable sea. As the renormalization scale gets smaller, the resolution decreases, sea quarks get recombined into gluons, and gluons are reabsorbed by quarks. Indeed, one can speculate that at very low renormalization scales, virtually all of the sea and most of the glue can be reabsorbed into the valence quarks, so that one is left with a proton composed of three “constituent” quarks bound together by some residual interaction, a quark model[2]! While this argument says nothing about which of the great variety of quark models describes the nucleon, it provides a well-defined prescription for testing models

against high energy data. One simply calculates quark distribution functions using the model and then uses the renormalization group to evolve the result from the quark model scale, μ_0^2 , to a scale relevant to experiment.

The first step is to calculate the quark distribution at the quark model scale. For unpolarized scattering, the spin averaged quark distributions can be written, in the nucleon rest frame, as[3]

$$\begin{aligned} q_i(x) &= \frac{1}{4\pi} \int d\xi^- e^{iq^+\xi^-} \langle N | \bar{\psi}_i(\xi^-) \gamma^+ \mathbf{P}_+(\xi^-) \psi_i(0) | N \rangle_{LC} \\ \bar{q}_i(x) &= -\frac{1}{4\pi} \int d\xi^- e^{iq^+\xi^-} \langle N | \bar{\psi}_i(0) \gamma^+ \mathbf{P}_-(\xi^-) \psi_i(\xi^-) | N \rangle_{LC}, \end{aligned} \tag{1}$$

where $q^+ = -Mx/\sqrt{2}$, with x the Bjorken scaling variable, $\psi_i(\bar{\psi}_i)$ are quark field operators of flavor i , $\gamma^+ = (\gamma^0 + \gamma^3)/\sqrt{2}$ is a Dirac gamma matrix, $\mathbf{P}_\pm(\xi^-)$ is a path ordered exponential, $\exp[\pm ig_s \int_0^{\xi^-} A^\pm(\eta^-) d\eta^-]$ with g_s the strong coupling constant, that insures the gauge invariance of the operator, and the subscript LC denotes the light cone condition on ξ , namely $\vec{\xi}_\perp = \xi^+ = 0$. The approach we shall take has been described in detail in Refs. 3-5 for several models, and consists of a straightforward evaluation of the matrix elements in Eq. 1, using a Peierls-Yoccoz momentum eigenstate[8] to describe the nucleon in its rest frame. While other approaches exist in the literature[9], our belief is that the current method maintains closer contact with the quark model in question, and is more easily generalized to systems

of more than one nucleon, which we will examine elsewhere.

Phenomenologically, a problem arises when the quark distributions are evaluated using the unperturbed, SU(4) symmetric nucleon wavefunction. As a result of the symmetry, the u and d valence quark distributions have the same functional form, and consequently $d_v(x)/u_v(x) = 1/2$ for all x . Experimentally, it is well known that the ratio decreases as x increases, and is thought to vanish linearly as $x \rightarrow 1$. The missing ingredient, it seems, is a mechanism for SU(4) breaking.

Fortunately, two mechanisms immediately present themselves. The first is the idea that the nucleon is surrounded by a cloud of virtual mesons, mainly pions, and that the pions contribute a flavor asymmetry to both the valence and sea distributions[10]. While this idea is certainly worthy of study, it is not the subject of the current effort. Instead, we shall concentrate on the second mechanism for SU(4) breaking, the color magnetic interaction.

The essential idea here is that the spin-spin interaction results in a dependence of the quark wavefunctions on the spin state of the other quarks that reside in the nucleon. Since, in the naive SU(4) wavefunction of the nucleon, the spin is unevenly distributed among the different flavor quarks, the spin dependence of the color magnetic interaction is transmuted into a flavor dependence of the spin averaged quark wavefunctions. Put into the calculation of quark distributions, this flavor dependence in the wavefunctions turns into a flavor dependence of the quark momentum distributions.

In this paper, we calculate the color magnetic corrections to the valence distributions of the nucleon using the Los Alamos Model Potential(LAMP). In the next section, we briefly describe the LAMP, and calculate its quark distributions in the absence of color magnetic effects. We then begin the calculation of the color magnetic corrections, separating out contributions to the valence distributions involving two and three body effects, respectively. We then describe the contributions of the gauge correction to the valence distributions, and how they may be partially resummed to obtain improved distributions. Finally, we shall evolve the full expressions for the corrected u and d distributions to high Q^2 , where they will be compared with experiment, and discuss the results.

2 Two and Three body Corrections

In order to set the stage for the forthcoming $\mathcal{O}(\alpha_s)$ calculations, we begin by calculating the unperturbed quark distributions for the LAMP model[11]. The model consists of three massless quarks, bound together in a linear scalar potential,

$$V(r) = V_0(r - r_0), \quad (2)$$

with parameters $V_0 = 0.9 \text{ GeV/fm}$ and $r_0 = 0.57 \text{ fm}$. Wavefunctions and single particle energies are obtained by solving the Dirac equation for this potential. As is usual for quark models, the potential parameters are chosen to generate the average of the nucleon and delta masses, and the color magnetic

interaction is invoked to generate the nucleon-delta splitting.

In Ref. 5, the matrix elements in Eq. 1 are evaluated, assuming that the nucleon is described by a Peierls-Yoccoz momentum eigenstate,

$$|N, P = \vec{0}\rangle = \lambda \int d^3a |N, \vec{R}_{cm} = \vec{a}\rangle, \quad (3)$$

where λ is a constant required to covariantly normalize the state, and $|N \vec{R}_{cm} = \vec{a}\rangle$ denotes the unprojected state with center fixed at $\vec{R}_{cm} = \vec{a}$. If one assumes that the time dependence of the quark field operators is well approximated by the single particle eigenvalue of the Dirac equation, the expression for the valence quark distribution is given by

$$q_V^i(x) = \frac{MN_i}{\pi V} \left[\int_{|k_-|}^{\infty} dk G(k) (t_0^2(k) + t_1^2(k) + \frac{2k_-}{k} t_0(k) t_1(k)) \right. \\ \left. + [k_- \rightarrow k_+] \right], \quad (4)$$

where

$$G(k) = \int r dr \sin kr \Delta^2(r) EB(r), \\ V = \int r^2 dr \Delta^3(r) EB(r), \\ t_0(k) = \int r^2 dr j_0(kr) u(r), \\ t_1(k) = \int r^2 dr j_1(kr) v(r), \\ \Delta(r) = \int d^3z \psi_0^\dagger(\vec{z} - \vec{r}) \psi_0(\vec{z}), \quad (5)$$

with $\psi_0(\vec{r})$ the ground state, single quark wavefunction, with upper and lower components $u(r)$ and $i\vec{\sigma} \cdot \vec{r}v(r)/r$, $k_\pm = \omega \pm Mx$, with ω the single particle

energy, and $EB(r) = \langle EB, \vec{R}_{cm} = r | EB, \vec{R}_{cm} = \vec{0} \rangle$ is the matrix element between two “empty bags” separated by a distance r , which accounts for the dynamics of the confining forces. In the LAMP picture, the dynamics of the confining fields are not specified, so we have no guidance on the proper choice for this function[6]. For the remainder of this paper, we assume $EB(r) \approx \text{const.}$ The resulting distribution is plotted, along with those of the MIT[4] and soliton[7] bag models in figure 1. The LAMP distribution lies between the MIT bag, where the valence quarks carry all of the momentum at the bag scale, and the soliton bag, where roughly a quarter of the proton’s momentum is carried by the confining degrees of freedom. We expect, therefore, that the quark model scale, μ_0^2 , from which we need to evolve the LAMP distribution in order to compare with experiment, will lie somewhat below the .6 GeV² found in reference 5 for the soliton bag. If we vary our assumption of a constant $EB(r)$, it is easy to generate distributions that interpolate continuously between the curve shown in figure 1 and curves resembling the soliton bag distribution. Correspondingly, the scale μ_0^2 will increase to a value in the neighborhood of .6 GeV².

Before calculating the color magnetic corrections to Eq. 4, we must engage in a brief digression on the questions of renormalization prescriptions and gauge choices. As it stands, Eq. 4 represents the results of a QCD inspired model of the structure of the nucleon. If the model were, in fact, a solution of the QCD Hamiltonian in some calculational scheme, the choice of

gauge and renormalization scheme would be specified, and the quark model wavefunctions would be unambiguously defined. Unfortunately, there is no known scheme for generating the LAMP(or any other) model directly from QCD, and consequently the gauge and renormalization schemes, if any, that produce the wavefunctions of reference 9 are unspecified. In order to completely specify the model, it is necessary to postulate that the model wavefunctions are calculated in a *particular* gauge, with a *particular* choice of renormalization scheme. As these choices are necessarily ad hoc, it follows that different choices will, in general, result in different models, and necessarily different predictions for physical observables. For the purposes of this paper, we shall choose the \overline{MS} renormalization scheme, and $A^0 = 0$ gauge.

The strategy we shall employ for evaluating the color magnetic corrections to the valence quark distributions is a straightforward application of perturbation theory, augmented by a closure approximation whose scale is the mean energy of the gluon in the nucleon. The interaction Hamiltonian is given by

$$H_I = g_s \int d^3r \vec{A}_a(\vec{r}, t) \vec{J}^a(\vec{r}, t), \quad (6)$$

where g_s is the strong coupling constant, $\vec{A}_a(\vec{r}, t)$ is the gauge field, and $\vec{J}^a(\vec{r}, t) = \bar{\psi}(\vec{r}, t) \frac{\lambda^a}{2} \vec{\gamma} \psi(\vec{r}, t)$ is the colored quark current operator, with $\frac{\lambda^a}{2}$ an SU(3) generator, and $\vec{\gamma}$ a Dirac matrix. To leading order in g_s , the nucleon wavefunction is given by

$$|N, \vec{P} = \vec{0}\rangle = Z^{-\frac{1}{2}} [|N_0, \vec{P} = 0\rangle + \frac{1}{E_0 - H_0} H_I |N_0, \vec{P} = 0\rangle], \quad (7)$$

where Z is a wavefunction renormalization constant, and E_0 is the ground state energy before correction. The energy shift is given by

$$\delta E = \frac{1}{2E_0 V_\infty} \langle N_0, \vec{P} = \vec{0} | H_I \frac{1}{E_0 - H_0} H_I | N_0, \vec{P} = \vec{0} \rangle, \quad (8)$$

with V_∞ the volume of space, and the renormalization constant Z is

$$Z = 1 + \frac{1}{2E_0 V_\infty} \langle N_0, \vec{P} = \vec{0} | H_I \frac{1}{(E_0 - H_0)^2} H_I | N_0, \vec{P} = \vec{0} \rangle \quad (9)$$

Inserting a complete set of gluon states, and a complete set of colored, 3 quark states, and using the fact that the unperturbed Hamiltonian from Eq. 2 is independent of color, the energy denominators may be seen to be given by

$$H_0 - E_0 = E_R(\vec{k}) + \omega(\vec{k}) - E_0, \quad (10)$$

where \vec{k} is the gluon momentum, $\omega(\vec{k})$ its energy, and $E_R(\vec{k})$ is the energy of an excited 3 quark state R with momentum $-\vec{k}$. In order to parallel the assumptions regarding the gluon propagator in reference 9, and to facilitate evaluation of the numerical integrations to come, we make the ansatz that $\omega(\vec{k}) = \mu e^{k^2/2\mu^2}$, where μ is an effective gluon mass taken to be 400 MeV. The energy denominator is then given by

$$H_0 - E_0 = \mu e^{k^2/2\mu^2} (1 + e^{-k^2/\mu^2} (E_R(\vec{k}) - E_0)/\mu). \quad (11)$$

A reasonable expectation is that the OGE process couples dominantly to the ground state, and that the relevant gluon momenta are small (on the order of $1/R_p$), so that the recoil energy of the massive nucleon is small, and

the second term in parentheses can be neglected. With this assumption, the energy denominator becomes independent of the intermediate three quark excited state, and closure may be invoked to perform the sum over excited states. The results of equations 7-9 keep the same form, except that the factors $E_0 - H_0$ are replaced by the constant $\omega(\vec{k})$.

Setting $\mathbf{P}_\pm = 1$, the $\mathcal{O}(\alpha_s)$ corrections to eq. 1 may be obtained using eq. 7. The result is

$$\begin{aligned} \delta q(x) = & -Z^{-1} \frac{1}{\mu^2} \int d\xi^- e^{-iMx\xi^-/\sqrt{2}} \int d^3r d^3r' G(\vec{r} - \vec{r}') \\ & \times \langle N_0, \vec{P} = \vec{0} | \vec{J}_a(\vec{r}) \bar{\psi}(\xi^-) \gamma^+ \psi(0) \vec{J}_a(\vec{r}') | N_0, \vec{P} = \vec{0} \rangle_{LC} \\ & + (Z^{-1} - 1) q^0(x), \end{aligned} \quad (12)$$

where $J_a(\vec{r}) = \bar{\psi}(\vec{r}) \frac{\lambda_a}{2} \psi(\vec{r})$, $G(\vec{r}) = A e^{-\mu^2 r^2/4}$ is the effective gluon propagator in the closure approximation, with $A \propto \alpha_s = \frac{g_s^2}{4\pi}$ an overall strength parameter chosen to reproduce the nucleon-delta mass splitting, and $q^0(x)$ is the unperturbed quark distribution of eq. 4.

2.1 Two-Body Correction

In general, eq. 12 gives rise to three types of terms which may be classified by the number of valence quarks acted on by the three currents appearing in the matrix element. The “one-body” terms, in which all three currents act on the same quark, represent self interactions and are traditionally ignored in quark model calculations. Here, these corrections will tend to mimic the effects of QCD evolution, and may be absorbed into a redefinition of the

scale μ_0^2 where the evolution begins. Since a precise determination of this parameter is not the purpose of the present effort, we shall honor tradition and neglect the one-body terms.

Next, there are the “two-body” terms, which arise when a gluon is in the process of being exchanged between two valence quarks when one of these quarks is struck by an external probe, such as a photon. Finally, there are three-body terms, in which the two spectator quarks exchange a gluon while the struck quark interacts with the external probe. These terms cannot be absorbed into a redefinition of μ_0 , since as we shall see, they are dependent on the isospin of the struck quark.

Using fermion anticommutation relations, the two and three- body corrections to the valence quark distributions can be extracted in a straightforward manner. The result for the two-body correction is

$$\begin{aligned} \delta q_\alpha^{2b}(x) = & \frac{\lambda^2}{\mu^2} \sum_{\beta \neq \alpha} \langle \frac{\lambda_a^\alpha \lambda^{a\beta}}{4} \rangle \int \frac{d^3 k_1 d^3 k_2}{(2\pi)^6} (2\pi) \delta(\frac{Mx - \omega - \hat{z} \cdot \vec{k}_1}{\sqrt{2}}) \\ & \times \left[\vec{Z}_\beta(\vec{k}_1, \vec{k}_2) \cdot \bar{\psi}_{0\alpha}(\vec{k}_2) \vec{\gamma} \gamma^0 \gamma^+ \psi_{0\alpha}(\vec{k}_1) + (\vec{k}_1 \leftrightarrow \vec{k}_2, \gamma^+ \leftrightarrow \vec{\gamma}) \right], \end{aligned} \quad (13)$$

where α denotes the struck quark, β the spectator with whom the gluon is exchanged, $\psi_0(\vec{k})$ is the quark momentum space wavefunction, the brackets denote an average over the color wavefunction of the proton, and

$$\vec{Z}_\beta(\vec{k}_1, \vec{k}_2) = \int d^3 r_1 d^3 r_2 e^{i(\vec{k}_1 \cdot \vec{r}_1 + \vec{k}_2 \cdot \vec{r}_2)} \vec{Z}_\beta(\vec{r}_1, \vec{r}_2) \Delta(\vec{r}_1 - \vec{r}_2) EB(\vec{r}_1 - \vec{r}_2), \quad (14)$$

with

$$\vec{Z}_\beta(\vec{r}_1, \vec{r}_2) = \int d^3x G(\vec{x}) \bar{\psi}_{0\beta}(\vec{x} - \vec{r}_1) \vec{\gamma} \psi_{0\beta}(\vec{x} - \vec{r}_2). \quad (15)$$

In order to extract the physically relevant u and d quark distributions, we must average over the spins in eqs. 13-15. To do the average, we must separate the spin dependence of the quark wavefunctions appearing in eq. 13. For s-wave quark wavefunctions, we get

$$\begin{aligned} \bar{\psi}_{0\alpha}(\vec{k}_1) \vec{\gamma} \gamma^0 \gamma^+ \psi_{0\alpha}(\vec{k}_2) &= \vec{k}_1 C_{01}(k_2, k_1) - \vec{k}_2 C_{01}(k_1, k_2) \\ &\quad + i((\vec{k}_2 \times \vec{\sigma}_\alpha) C_{01}(k_1, k_2) + (\vec{k}_1 \times \vec{\sigma}_\alpha) C_{01}(k_2, k_1)) \\ &\quad + \hat{z}(C_{00}(k_1, k_2) - (\vec{k}_1 \cdot \vec{k}_2 + i\vec{k}_2 \cdot (\vec{k}_1 \times \vec{\sigma}_\alpha)) C_{11}(k_1, k_2)) \\ &\quad + i\hat{z} \times (\vec{\sigma}_\alpha(C_{00}(k_1, k_2) + \vec{k}_1 \cdot \vec{k}_2 C_{11}(k_1, k_2)) \\ &\quad - (\vec{k}_2 \vec{k}_1 \cdot \vec{\sigma}_\alpha + \vec{k}_1 \vec{k}_2 \cdot \vec{\sigma}_\alpha + i(\vec{k}_1 \times \vec{k}_2)) C_{11}(k_1, k_2))), \end{aligned} \quad (16)$$

where

$$C_{ij}(k_1, k_2) = \frac{4\pi}{\sqrt{2}} \frac{t_i(k_1) t_j(k_2)}{k_1^i k_2^j}, \quad (17)$$

and $(i, j) \in (0, 1)$ as defined in eq. 5. The function $\vec{Z}_\beta(\vec{k}_1, \vec{k}_2)$ can be decomposed similarly,

$$\begin{aligned} \vec{Z}_\beta(\vec{k}_1, \vec{k}_2) &= \vec{k}_1 Z_\beta^1(\vec{k}_1, \vec{k}_2) + \vec{k}_2 Z_\beta^2(\vec{k}_1, \vec{k}_2) \\ &\quad + i((\vec{k}_1 \times \vec{\sigma}_\beta) Z_\beta^3(\vec{k}_1, \vec{k}_2) + (\vec{k}_2 \times \vec{\sigma}_\beta) Z_\beta^4(\vec{k}_1, \vec{k}_2)), \end{aligned} \quad (18)$$

where the $Z_\beta^i(\vec{k}_1, \vec{k}_2)$ are scalar functions of \vec{k}_1 and \vec{k}_2 , evaluated semi-analytically using eq. 15 and the six gaussian fit to the quark wavefunctions of reference 9.

Once the integrations are performed, there are only three vectors left, $\hat{z}, \vec{\sigma}_\alpha$ and $\vec{\sigma}_\beta$, and the two-body correction takes the form

$$\delta q_\alpha^{2b}(x) = f^{2b}(x) + \sum_{\beta \neq \alpha} (\vec{\sigma}_\alpha^i \vec{\sigma}_\beta^j T_{ij}^{2b}(x), \quad (19)$$

where $T_{ij}^{2b}(x)$ is a tensor defined in terms of the quark wavefunctions.

Using rotational arguments, it is easy to demonstrate that only the trace of T will contribute to the spin independent quark distribution. Hence, the functions $f^{2b}(x)$ and $T_{ii}(x)$ may be written as

$$\begin{aligned} f^{2b}(x) = & -\frac{16Z^{-1}\lambda^2}{3\mu^2} \int \frac{d^3k_1 d^3k_2}{(2\pi)^5} \delta(Mx - \omega - \hat{z} \cdot \vec{k}_1) \\ & \times \left[Z^2(\vec{k}_1, \vec{k}_2) \left(k_1^2 C_{10}(k_1, k_2) + \vec{k}_1 \cdot \vec{k}_2 C_{01}(k_1, k_2) \right. \right. \\ & \quad \left. \left. + \hat{z} \cdot \vec{k}_1 (C_{00}(k_1, k_2) + \vec{k}_1 \cdot \vec{k}_2 C_{11}(k_1, k_2)) \right) \right. \\ & \left. + Z^1(\vec{k}_1, \vec{k}_2) \left(\vec{k}_1 \cdot \vec{k}_2 C_{10}(k_1, k_2) + k_2^2 C_{01}(k_1, k_2) \right. \right. \\ & \quad \left. \left. + \hat{z} \cdot \vec{k}_1 \left(\frac{\vec{k}_1 \cdot \vec{k}_2}{k_1^2} C_{00}(k_1, k_2) + k_2^2 C_{11}(k_1, k_2) \right) \right) \right], \end{aligned} \quad (20)$$

and,

$$\begin{aligned} T_{ii}^{2b}(x) = & -\frac{32Z^{-1}\lambda^2}{3\mu^2} \int \frac{d^3k_1 d^3k_2}{(2\pi)^5} \delta(Mx - \omega - \hat{z} \cdot \vec{k}_1) \\ & \times \left[Z^4(\vec{k}_1, \vec{k}_2) \left(k_1^2 C_{10}(k_1, k_2) - \vec{k}_1 \cdot \vec{k}_2 C_{01}(k_1, k_2) \right. \right. \\ & \quad \left. \left. + \hat{z} \cdot \vec{k}_1 (C_{00}(k_1, k_2) - \vec{k}_1 \cdot \vec{k}_2 C_{11}(k_1, k_2)) \right) \right. \\ & \left. + Z^3(\vec{k}_1, \vec{k}_2) \left(\vec{k}_1 \cdot \vec{k}_2 C_{10}(k_1, k_2) - k_2^2 C_{01}(k_1, k_2) \right) \right] \end{aligned}$$

$$+\hat{z} \cdot \vec{k}_2(C_{00}(k_1, k_2) - \vec{k}_1 \cdot \vec{k}_2 C_{11}(k_1, k_2))\Big], \quad (21)$$

Assuming that the unperturbed proton wavefunction is SU(4) symmetric, the spin sums are given by

$$\sum_{\alpha \neq \beta} \vec{\sigma}_\alpha \cdot \vec{\sigma}_\beta \frac{(1 \pm \tau_{3\alpha})}{2} = -3 \pm 1, \quad (22)$$

so that the two-body corrections to the physical quark distributions are given by

$$\delta u^{2b}(x) = 2f^{2b}(x) - 2T_{ii}^{2b}(x) \quad (23)$$

$$\delta d^{2b}(x) = f^{2b}(x) - 4T_{ii}^{2b}(x). \quad (24)$$

An interesting feature of this result is that the d quark distribution is twice as sensitive to color magnetic effects than the u distribution. On physical grounds, this is understandable, since the two spectator u quarks are necessarily in a spin 1 state in the SU(4) wavefunction, while the spectator ud pair is most likely to be found in a spin 0 state. Numerically, the color electric correction, $f^{2b}(x)$, is quite small and is neglected in the following.

The correction to the antiquark distribution is obtained from eq. 13 by replacing $\omega \leftrightarrow -\omega$ and including an overall minus sign.

2.2 Three Body Correction

After separating out the one and two-body terms from equation 12, only the three-body term remains. Physically, this correction accounts for the mod-

ified structure of the spectator diquark, and is expressed as a modification of the recoil function $G(k)$ appearing in equation 4. In terms of the quark wavefunctions the three body correction is given by

$$\begin{aligned} \delta q_V^{3b}(x) = & \frac{Z^{-1}\lambda^2}{\mu^2} \sum_{\alpha \neq \beta \neq \epsilon} \langle \frac{\lambda_a^\epsilon \lambda_a^\beta}{4} \rangle \int \frac{d^3 k}{(2\pi)^3} (2\pi) \delta(\frac{Mx - \omega - \hat{z} \cdot \vec{k}}{\sqrt{2}}) \\ & \times \bar{\psi}_{0\alpha}(\vec{k}) \gamma^+ \psi_{0\alpha}(\vec{k}) F_{\beta\epsilon}(-\vec{k}), \end{aligned} \quad (25)$$

where the function $\tilde{F}_{\beta\epsilon}(\vec{k})$ is given by

$$\tilde{F}_{\beta\epsilon}(\vec{k}) = \int d^3 y e^{i\vec{k} \cdot \vec{y}} F_{\beta\epsilon}(\vec{y}) EB(y), \quad (26)$$

with

$$F_{\beta\epsilon}(\vec{z}) = \int \int d^3 r_1 d^3 r_2 G(\vec{r}_1 - \vec{r}_2) \bar{\psi}_{0\beta}(\vec{r}_1) \vec{\gamma} \psi_{0\beta}(\vec{r}_1 - \vec{z}) \cdot \bar{\psi}_{0\epsilon}(\vec{r}_2) \vec{\gamma} \psi_{0\epsilon}(\vec{r}_2 - \vec{z}). \quad (27)$$

Just like the two-body correction, $F_{\beta\epsilon}(\vec{z})$ can be decomposed into a scalar function, $f^{3b}(z)$ and the trace of a tensor $T_{ij}^{3b}(\vec{z})$,

$$F_{\beta\epsilon}(\vec{z}) = f^{3b}(z) + \frac{\vec{\sigma}_\beta \cdot \vec{\sigma}_\epsilon}{3} T_{ii}^{3b}(z), \quad (28)$$

where

$$\begin{aligned} f^{3b}(z) = & -2 \int \int d^3 r_1 d^3 r_2 G(\vec{r}_1 - \vec{r}_2) \left[v(r_1) v(r_2) u(\vec{r}_1 - \vec{z}) u(\vec{r}_2 - \vec{z}) \hat{r}_1 \cdot \hat{r}_2 \right. \\ & \left. - v(r_1) v(\vec{r}_2 - \vec{z}) u(r_2) v(\vec{r}_2 - \vec{z}) \hat{r}_1 \cdot \frac{(\vec{r}_2 - \vec{z})}{|\vec{r}_2 - \vec{z}|} \right], \end{aligned} \quad (29)$$

and,

$$\begin{aligned} T_{ii}^{3b}(z) = & 4 \int \int d^3 r_1 d^3 r_2 G(\vec{r}_1 - \vec{r}_2) \left[v(r_1) v(r_2) u(\vec{r}_1 - \vec{z}) u(\vec{r}_2 - \vec{z}) \hat{r}_1 \cdot \hat{r}_2 \right. \\ & \left. + v(r_1) v(\vec{r}_2 - \vec{z}) u(r_1) v(\vec{r}_2 - \vec{z}) \hat{r}_1 \cdot \frac{(\vec{r}_2 - \vec{z})}{|\vec{r}_2 - \vec{z}|} \right], \end{aligned} \quad (30)$$

where u and v are, again, the upper and lower components of the single quark wavefunctions as in eq. 5.

The spin sum, including the projection onto quark isospin, is given by

$$\sum_{\alpha \neq \beta \neq \epsilon} \frac{(1 \pm \tau_{3\alpha})}{2} \vec{\sigma}_\beta \cdot \sigma_\epsilon = \frac{-3 \mp 5}{2} \quad (31)$$

so that the three body correction is given by

$$\begin{aligned} \delta u^{3b}(x) &= 2\delta q_E^{3b}(x) - 4\delta q_M^{3b}(x) \\ \delta d^{3b}(x) &= \delta q_E^{3b}(x) + \delta q_M^{3b}(x), \end{aligned} \quad (32)$$

with

$$\begin{aligned} \delta q_E^{3b}(x) &= \frac{Z^{-1}\lambda^2}{\mu^2} \sum_{\alpha \neq \beta \neq \epsilon} \langle \frac{\lambda_a^\epsilon \lambda_a^\beta}{4} \rangle \int \frac{d^3k}{(2\pi)^3} (2\pi) \delta(\frac{Mx - \omega - \hat{z} \cdot \vec{k}}{\sqrt{2}}) \\ &\quad \times \bar{\psi}_{0\alpha}(\vec{k}) \gamma^+ \bar{\psi}_{0\alpha}(\vec{k}) \tilde{f}^{3b}(-\vec{k}), \end{aligned} \quad (33)$$

and

$$\begin{aligned} \delta q_M^{3b}(x) &= \frac{Z^{-1}\lambda^2}{\mu^2} \sum_{\alpha \neq \beta \neq \epsilon} \langle \frac{\lambda_a^\epsilon \lambda_a^\beta}{4} \rangle \int \frac{d^3k}{(2\pi)^3} (2\pi) \delta(\frac{Mx - \omega - \hat{z} \cdot \vec{k}}{\sqrt{2}}) \\ &\quad \times \bar{\psi}_{0\alpha}(\vec{k}) \gamma^+ \bar{\psi}_{0\alpha}(\vec{k}) \tilde{T}_{ii}^{3b}(-\vec{k}), \end{aligned} \quad (34)$$

and where $\tilde{f}^{3b}(\vec{k})$, $\tilde{T}_{ii}^{3b}(\vec{k})$ are the Fourier transforms of $EB(z)f^{3b}(\vec{z})$ and $EB(z)T_{ii}^{3b}(\vec{z})$, respectively. Again the relative weights appearing in the eq. 32 reflect the fact that a ud spectator pair is most likely to be in a spin zero state.

3 Gauge Correction

As discussed in the last section, a gauge and renormalization scheme must be assumed in order for quark model wavefunctions to be meaningfully defined. Correspondingly, the effect of the path-ordered exponential of eq. 1 must be included in order to obtain quark distributions that are gauge invariant. Expanding the path ordered exponential $P_{\pm}(\xi)$ to leading order in g_s , we obtain

$$\begin{aligned} \delta q_{gauge}(x) = & i\mu \int d\xi^- \int_0^{\xi^-} d\eta^- \langle N_0, \vec{P} = \vec{0} | \bar{\psi}(\xi^-) A^+(\eta^-) \psi(0) \\ & \times \int d^3r \vec{J}(\vec{r}) \cdot \vec{A}(\vec{r}) | N_0, \vec{P} = \vec{0} \rangle. \end{aligned} \quad (35)$$

Neglecting self interactions, this expression may be evaluated in the same fashion as the two and three body corrections. The result, after some manipulation, is given by

$$\begin{aligned} \delta q_{gauge}(x) = & \frac{16\lambda^2\sqrt{2}}{\mu M} \frac{d}{dx} \sum_{\alpha \neq \beta} \int_0^1 dy \int \frac{d^3P d^3q}{(2\pi)^5} \delta(Mx - \omega - \hat{z} \cdot q) \\ & \times \bar{\psi}_{\alpha}(y\vec{P} - \vec{q}) \gamma^+ \psi_{\alpha}((1-y)\vec{P} + \vec{q}) \hat{z} \cdot \vec{Z}_{\beta}(y\vec{P} - \vec{q}, (1-y)\vec{P} + \vec{q}). \end{aligned} \quad (36)$$

The spin structure of the gauge correction is identical to that of the two-body correction, so we may immediately write

$$\begin{aligned} \delta u_{gauge}(x) &= \frac{d}{dx} (2f^{gauge}(x) + 2T_{ii}^{gauge}(x)) \\ \delta d_{gauge}(x) &= \frac{d}{dx} (f^{gauge}(x) + 4T_{ii}^{gauge}(x)), \end{aligned} \quad (37)$$

where

$$\begin{aligned}
f^{gauge}(x) = & \frac{16\lambda^2}{\mu M} \int_0^1 dy \int \frac{d^3 P d^3 q}{(2\pi)^5} \delta(Mx - \omega - \hat{z} \cdot q) \\
& \times \left[Z^1(\vec{k}_2, \vec{k}_1) \hat{z} \cdot \vec{k}_2 (\hat{z} \cdot \vec{k}_2 C_{10}(k_1, k_2) + \hat{z} \cdot \vec{k}_1 C_{01}(k_1, k_2) \right. \\
& \quad \left. + (C_{00}(k_1, k_2) + \vec{k}_1 \cdot \vec{k}_2) C_{11}(k_1, k_2)) \right. \\
& \quad \left. + Z^2(\vec{k}_2, \vec{k}_1) \hat{z} \cdot \vec{k}_1 (\hat{z} \cdot \vec{k}_2 C_{10}(k_1, k_2) + \hat{z} \cdot \vec{k}_1 C_{01}(k_1, k_2) \right. \\
& \quad \left. + (C_{00}(k_1, k_2) + \vec{k}_1 \cdot \vec{k}_2) C_{11}(k_1, k_2)) \right],
\end{aligned} \tag{38}$$

$$\begin{aligned}
T_{ii}^{gauge}(x) = & -\frac{16\lambda^2}{3\mu M} \int_0^1 dy \int \frac{d^3 P d^3 q}{(2\pi)^5} \delta(Mx - \omega - \hat{z} \cdot \vec{q}) \\
& \times \left[Z^3(\vec{k}_2, \vec{k}_1) ((\vec{k}_1 \cdot \vec{k}_2 - \hat{z} \cdot \vec{k}_1 \hat{z} \cdot \vec{k}_2) C_{01}(k_1, k_2) \right. \\
& \quad - (k_2^2 - (\hat{z} \cdot \vec{k}_2)^2) C_{10}(k_1, k_2) + (\hat{z} \cdot \vec{k}_2 \vec{k}_1 \cdot \vec{k}_2 - k_2^2 \hat{z} \cdot \vec{k}_1) C_{11}(k_1, k_2)) \\
& \quad - Z^4(k_2, k_1) ((\vec{k}_1 \cdot \vec{k}_2 - \hat{z} \cdot \vec{k}_1 \hat{z} \cdot \vec{k}_2) C_{10}(k_1, k_2) \\
& \quad \left. - (k_1^2 - (\hat{z} \cdot \vec{k}_1)^2) C_{01}(k_1, k_2) - (k_1^2 \hat{z} \cdot \vec{k}_1 - \vec{k}_1 \cdot \vec{k}_2 \hat{z} \cdot \vec{k}_1) C_{11}(k_1, k_2)) \right]
\end{aligned} \tag{39}$$

with $\vec{k}_1 = (1 - y)\vec{P} + \vec{q}$ and $\vec{k}_2 = y\vec{P} - \vec{q}$.

The gauge factor in eq. 1 removes the unphysical, gauge dependent phase in the quark wavefunction, which produces a mismatch between the momentum fraction x calculated using simple Fourier transforms and the physical momentum carried by the struck quark. This shift in x is manifested by the derivative with respect to x appearing in the gauge correction. This pattern persists in higher orders, as the path ordered exponential is expanded

in powers of $g_s \frac{d}{dx}$, and suggests that the gauge correction may be partially resummed to obtain a quark distribution of the form

$$q^i(x) = (1 + \frac{ds_i(x)}{dx})q_0^i(x + s_i(x)), \quad (40)$$

where $q_0^i(x)$ is the zeroeth order quark distribution for quarks of flavor i . To leading order in α_s , the shift function $s_i(x)$ is given by

$$s_i(x) = \frac{(a_i f^{gauge}(x) + b_i T_{ii}^{gauge}(x))}{q_0^i(x)}, \quad (41)$$

with a_i and b_i the isospin dependent coefficients appearing in eq. 37. Since both α_s and the derivatives of quark distributions can be large in the context of quark models, the effects of the resummation can be important, particularly for the d distribution.

4 Results

The expressions for the gauge, two and three-body corrections to the valence distributions of the proton were evaluated using a Gaussian quadrature scheme to perform the multiple integrations appearing in eqs. 20,21,33,34,38, and 39. For the gauge and two-body corrections, the function $\vec{Z}_\beta(\vec{k}_1, \vec{k}_2)$ was evaluated semi-analytically by making use of the 6 gaussian fit to the LAMP coordinate space wavefunctions described in reference 9. After accounting for the rotational symmetries of the problem, there were 4,5, and 6 integrations to be performed numerically for the two-body, gauge and three-body corrections, respectively. Treating all color magnetic effects perturbatively,

we obtain the curves shown in Figs. 2 and 3 for the d and u valence distributions, respectively. Also shown are the distributions with no color magnetic corrections applied, and the curves obtained by either neglecting the gauge correction, or by resumming the gauge correction in the fashion described at the end of section 3.

As shown in Figs. 2 and 3, the general effect of CMI is to lower the momentum carried by the valence quarks. Physically, this makes sense, since the exchanged gluons will carry $p^+ > 0$, and consequently the momentum fraction carried by the struck quark must decrease. As advertised in the derivation of the two and three-body corrections, the d quark distribution is more sensitive to CMI than the u distribution, since the uu spectator pair is more likely to be found in a spin 1 state than the ud pair. Less encouraging is the fact that the Peierls-Yoccoz projection procedure is significantly less effective in producing corrected distributions with good support properties than it is for the unperturbed distributions. In particular, the two-body correction to the valence distributions has a non-negligible tail ($\approx .01$) in the region $x \approx 1$. This increase in the relative size of the tail may be understood in terms of a small admixture of higher mass eigenstates in the Peierl-Yoccoz projected nucleon wavefunction. Since these states are more massive, the quarks they contain are not constrained to carry momentum $p^+ < M/\sqrt{2}$ in the nucleon rest frame, and show up as a tail in the region $x \geq 1$. Since there are many excited states to couple to, the size of this unwanted component

of the wavefunction will tend to grow when a perturbation, such as the color magnetic interaction, is introduced[12]. A more serious problem, from a phenomenological point of view, is the fact that the gauge correction is comparable to the two and three-body corrections. This suggests that the d quark distribution will be quite sensitive to the choice of gauge used to define quark model wavefunctions. In principle, this provides an additional restriction on the models, but in practice the infinite variety of gauges may provide a means to reconcile virtually any model with the data.

The quark distributions are evolved from μ_0^2 to 15 GeV^2 using a finite element procedure to reconstruct the evolved distributions from their moments. A detailed description of this procedure will be given elsewhere[13]. The results for the d and u distributions are shown in Figs. 4 and 5, respectively, for μ_0^2 ranging from .2-.4 GeV^2 . Also shown are BEBC neutrino data at the same Q^2 [14]. In general, the data at large x favor a choice of the scale μ_0^2 between .2 and .3 GeV^2 , while at smaller x , the data lies below model predictions even at these small renormalization scales. This situation may reflect the uncertainty associated with modeling the sea distributions, which are used to analyze the data in this region, or it may reflect a shortcoming of the model, namely that too much momentum is carried by the valence quarks at the quark model scale. In the former case, we note that parametrizations of parton distributions based on larger data sets[15] generate larger valence distributions in the region of x in question. In the latter case, we have al-

ready noted that the momentum fraction carried by the valence quarks may be altered significantly by changing the spatial dependence of the empty bag matrix element, $EB(z)$.

In light of these facts, we conclude that color magnetic interactions provide a natural mechanism for producing the observed differences between the u and d valence quark distributions of the proton, and that the LAMP picture provides a reasonable quantitative description of the valence distributions once color magnetic effects are included.

Acknowledgements

This work was supported in part by the U.S. Department of Energy, Division of High Energy and Nuclear Physics, ER-23.

References

- [1] R.L. Jaffe and G.G. Ross, Phys. Lett. **93B**, 313(1980).
- [2] R.P. Feynman, private communication.
- [3] R.L. Jaffe, in *Relativistic Dynamics and Quark Nuclear Physics*, proceedings of the Los Alamos Summer school, 1985, edited by M.B. Johnson and A. Picklesimer, Wiley, New York, 1986.
- [4] C.J. Benesh and G.A. Miller, Phys. Rev. **D36**,1344(1987).
- [5] C.J. Benesh and G.A. Miller, Phys. Rev. **D38**,48(1988).

- [6] A picture of confinement due to scalar glueball exchange has been developed by T. Goldman and A.S. Goldhaber, LA-UR-93-?????. In this picture, the scalar fields dynamics allow the empty bag matrix element to be calculated in a fashion similar to the soliton bag model of E.G. Lubeck et. al., Phys. Rev. **D33**, 234(1986).
- [7] C.J. Benesh and G.A. Miller, Phys. Lett. **B215**,381(1988).
- [8] R.E. Peierls and J. Yoccoz, Proc. Phys. Soc. London **A70**, 381(1957).
J.J. Griffin and J.A. Wheeler, Phys. Rev. **108**, 311(1957).
- [9] A.J. Signal and A.W. Thomas, Phys. Rev. **D40**, 2832(1989),
A.W. Thomas, Nucl. Phys. **A532**, 177C (1991).
- [10] N.N. Nikolaev, Tokyo Report No. INS-Rep-539,1985 (unpublished). It is worth noting that the straightforward application of this approach to nuclei leads to enhanced antiquark distributions, in conflict with nuclear Drell-Yan data.
- [11] T. Goldman, K.R. Maltman, G.J. Stephenson Jr., and K.E. Schmidt, Nucl. Phys. **A481**, 621(1988).
- [12] We note that this problem with perturbation theory may be avoided if the model is quantized on the light cone, but then the unperturbed wavefunction becomes difficult to obtain.
- [13] C.J. Benesh, to be published.

- [14] H. Abramowicz et. al., Z. Phys. **C25**, 29(1984).
- [15] A.J. Buras, Rev. Mod. Phys. **52**, 199(1980).

Figure Captions

- Figure 1 - Valence quark distributions for the MIT bag, soliton bag, and LAMP at the quark model scale.
- Figure 2 - Valence distributions for the d quark at the quark model scale, including color magnetic corrections.
- Figure 3 - Valence distributions for the u quark at the quark model scale, including color magnetic corrections.
- Figure 4 - Valence distributions for the d quark, evolved to $Q^2=15$ GeV². Also shown are data from BEBC.
- Figure 5 - Valence distributions for the u quark, evolved to $Q^2=15$ GeV². Also shown are data from BEBC.